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Gauge symmetry and cosmological constant on a brane

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Abstract

We have examined the localization of gauge bosons on the three-brane embedded in 5D bulk space-time, and we find two kinds of branes on which both graviton and gauge bosons can be trapped. One is the dS brane with a positive cosmological constant, and the other is the one with zero cosmological constant or 4D Minkowski brane. Then, which brane is realized would depend on the observation of the cosmological constant of our world.

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1 Introduction

It is fascinating to regard our four dimensional world as a three-brane [1, 2] embedded in a higher dimensional space with extended codimensions. In the recent progress of superstring theory or M-theory, we could find an appropriate non-perturbative solution which could be connected to a realistic brane-world. However, there are many things to be cleared before arriving at such a solution.

The idea of three-brane involves many interesting aspects to approach several fundamental problems. Actually, it opened an alternative to the standard Kaluza-Klein (KK) compactification via the localization of the graviton in the 4D Minkowski space [2] and also in the 4D de Sitter space [3, 4, 5]. In order to confirm the idea of braneworld, it would be important to see the localization of the gauge bosons, which is an important force to construct our world, i.e. the Coulomb force. However, gauge bosons feel the warp factor in the bulk differently from the case of the graviton, and it is a difficult problem how to localize the gauge bosons on the brane.

In the case of RS model, where 4d cosmological constant λ is zero, some ideas for the localization of gauge bosons have been proposed (for example, see [7, 8]). In general, we can expect that the situation of this problem would be changed when a small λ is considered on the brane since the warp factor is changed in this case compared to the naive model. Actually, we could find the localization of a massive mode of a scalar in this case [9].

Our purpose is to extend this analysis to the case of vector fields to see the localization of bulk vector-fields as gauge bosons on the brane. The localization of graviton or massless scalar can not give any restriction to the magnitude of the λ , and we can see the localization of these fields in a wide range of the parameters in the theory. While, we point out that localization of gauge bosons intimately is related to the value of λ as shown in the analyses given here.

For a brane with a positive λ , the bulk gauge bosons are trapped on it. And we should notice the recent observation of small inflation of our universe which implies the existence of small positive λ .

At the same time, we propose another kind of brane proposed previously by one of us [8]. In this case, the bulk vector is massive and a mass term of the vector is also attached on the brane action. The origin of the latter term might be found as quantum corrections in the bulk [10]. In this case, we could find a gauge boson trapped on the brane with $\lambda = 0$ when we choose an appropriate relation between the bulk and brane-coupled mass terms.

In Section 2, we give our model in the bulk and a brief review of brane solutions used here. In Section 3, for the case including manifest bulk gauge symmetry, the localization of the massive and massless vectors on the brane is examined. In Section 4, for the case with the brane coupling, the vector localization is examined. Concluding remarks are given in the final section.

2 Brane model and solutions with cosmological constant

Here we set up our model by the following effective action,

$$S = S_{\text{gr}} + S_{\text{A}}. \quad (1)$$

The first term denotes the gravitational part,

$$S_{\text{gr}} = \frac{1}{2\kappa^2} \left\{ \int d^5 X \sqrt{-G} (R - 2\Lambda) + 2 \int d^4 x \sqrt{-g} K \right\} - \tau \int d^4 x \sqrt{-g}, \quad (2)$$

where K is the extrinsic curvature on the boundary, and τ represents the tension of the brane which is situated at $y = 0$ by imposing $y \leftrightarrow -y$ symmetry for action. The second part S_{A} denotes the action for the massive vector, which is denoted by $A_M(x, y)$, ($M = 0, \dots, 4$),

$$S_{\text{A}} = \int d^5 X \sqrt{-G} \left\{ -\frac{1}{4} G^{MN} G^{PQ} F_{MP} F_{NQ} - \frac{1}{2} (M^2 + c \delta(y)) G^{MN} A_M A_N \right\}, \quad (3)$$

where the parameter M and c denote the bulk mass of the vector and the coupling of the vector to the brane respectively. In the case of Randall-Sundrum solution with zero-cosmological constant, the massive vector can be trapped as a photon only when we take the parameter c as follows [8],

$$c = -2\mu \left(\sqrt{1 + \frac{M^2}{\mu^2}} - 1 \right) \equiv \hat{c}, \quad (4)$$

where $\mu = \sqrt{-\Lambda/6}$. This relation seems to be a kind of fine-tuning of the parameters in the theory. Here we extend the analysis to the case of $\lambda \geq 0$ to see the localization of vector.

We consider the vector-field fluctuations on the background given as a solution of (2). Here, the background configuration with non-zero λ is considered, and the Einstein equation is solved in the following metric,

$$ds^2 = A^2(y) \left\{ -dt^2 + a^2(t) \gamma_{ij}(x^i) dx^i dx^j \right\} + dy^2, \quad (5)$$

where the coordinates parallel to the brane are denoted by $x^\mu = (t, x^i)$, and y being the coordinate transverse to the brane. We restrict our interest here to the case of a Friedmann-Robertson-Walker type universe. Then, the three-dimensional metric γ_{ij} is described in Cartesian coordinates as

$$\gamma_{ij} = (1 + k \delta_{mn} x^m x^n / 4)^{-2} \delta_{ij}, \quad (6)$$

where the curvature indices $k = 0, 1, -1$ correspond to a flat, closed, or open universe respectively. In this brane-world, the effective cosmological constant λ on the brane is given as

$$\lambda = \kappa^4 \tau^2 / 36 + \Lambda / 6. \quad (7)$$

We notice that λ depends on both parameters τ and Λ . The solutions of non-zero λ used here are summarized below.

(I) $\lambda > 0$: When $\lambda > 0$, we obtain the solution for $a_0(t)$ which represents inflation of three dimensional space. Here we give the simple one, which is used hereafter, for the case of $k = 0$,

$$a_0(t) = e^{H_0 t}, \quad (8)$$

where the Hubble constant is represented as $H_0 = \sqrt{\lambda}$. We notice that $a_0(t)$ has nothing to do with the problem of localization and it depends only on the form of $A(y)$ as we will see below.

As for $A(y)$, we obtain the same solution for any value of $k = 0, \pm 1$. The solution of $A(y)$ depends on the sign of Λ even if the same positive λ is assigned.

(I-1) $\Lambda < 0$: For $\Lambda < 0$, $A(y)$ is solved as

$$A(y) = \frac{\sqrt{\lambda}}{\mu} \sinh[\mu(y_H - |y|)] \quad (9)$$

$$\sinh(\mu y_H) = \mu/\sqrt{\lambda}. \quad (10)$$

where y_H represents the position of the horizon in the bulk. This solution and others are normalized at $y = 0$, as $A(0) = 1$. And all the background configurations are taken to be Z_2 symmetric with respect to the reflection, $y \rightarrow -y$.

(I-2) $\Lambda > 0$: When Λ is positive, the solution for $a_0(t)$ is the same as above, but $A(y)$ is different. One has

$$A(y) = \frac{\sqrt{\lambda}}{\mu_d} \sin[\mu_d(y_H - |y|)], \quad (11)$$

$$\mu_d = \sqrt{\Lambda/6}, \quad \sin(\mu_d y_H) = \mu_d/\sqrt{\lambda}. \quad (12)$$

These two configurations represent dS_4 -branes embedded in the bulk at $y = 0$.

(II) $\lambda < 0$: When $\lambda < 0$, we obtain

$$a_{-1}(t) = \frac{1}{H} \sin(Ht), \quad H = \sqrt{-\lambda} \quad (13)$$

and $k = -1$. The AdS_4 -brane is obviously obtained for $\Lambda < 0$ because of (7). And $A(y)$ is given as

$$A(y) = \frac{H}{\mu} \cosh[\mu(y_H - |y|)] \quad (14)$$

$$\cosh(\mu y_H) = \mu/H. \quad (15)$$

Note that the bulk has no horizon in this case in spite of the letter y_H used above. AdS_4 brane with $\lambda < 0$ can not be regarded as our world from the viewpoint of brane-world since the general coordinate invariance is lost [12]. However, we consider this case from the theoretical viewpoint.

3 Vector localization: $c = 0$ case

Here we concentrate our attention on the behaviors of the vector-field fluctuation around the background (5) by extending the previous analysis given in [8].

At first, we examine this problem for the case of $c = 0$ in the action (3). It contains the case where bulk gauge symmetry is manifest as a limit of $M = 0$. The field equation of A_M is given as,

$$\frac{1}{\sqrt{-G}}\partial_A[\sqrt{-G}(G^{AB}G^{CD} - G^{AC}G^{BD})\partial_B A_C] - M^2 G^{DB} A_B = 0. \quad (16)$$

This equation can be solved by using the identity obtained in the massive vector theory,

$$\partial_A(\sqrt{-G}G^{AB}A_B) = 0, \quad (17)$$

where we should notice that for $M = 0$, we interpret Eq.(17) as the gauge condition. We solve these equations by rewriting the five components of A_M as (A_t, A_i^T, A_i^L, A_y) , where the 3D transverse part is defined as $\nabla_i A_T^i = 0$ and A_i^L denotes the longitudinal component.

The next step is to expand the fields in terms of the four-dimensional mass eigenstates:

$$A_M(x, y) = \int dm \tilde{a}_M(m, t, x^i) \phi_M(m, y), \quad (18)$$

where the mass m is defined for each $\tilde{a}_M(m, t, x^i)$ as shown below. For A_i^T , we obtain,

$$(\partial_y^2 + 2\frac{A'}{A}\partial_y + \frac{m^2}{A^2})\phi_i^T = M^2\phi_i^T, \quad (19)$$

$$(-\partial_t^2 - \frac{\dot{a}_0}{a_0}\partial_t + \frac{\partial_i^2}{a_0^2})\tilde{a}_i^T = m^2\tilde{a}_i^T, \quad (20)$$

where $' = d/dy$ and $\dot{} = d/dt$. In deriving the above two equations, Eq.(17) is not used. Since Eq.(17) give a relation among A_y , A_μ^L and A_t , it is used below for the equations for those components. The second equation (20) gives a definition of the 4D mass m for the transverse components, and this definition is not common to other components, which are shown below. However we notice that they coincide when $a_0 = 1$. The equations for other components can be written as follows:

For A_y ,

$$(\partial_y^2 + 6\frac{A'}{A}\partial_y + 4[\frac{A''}{A} + (\frac{A'}{A})^2] + \frac{m^2}{A^2})\phi_y = M^2\phi_y, \quad (21)$$

$$(-\partial_t^2 - 3\frac{\dot{a}_0}{a_0}\partial_t + \frac{\partial_i^2}{a_0^2})\tilde{a}_y = m^2\tilde{a}_y, \quad (22)$$

For A_t ,

$$(\partial_y^2 + 2\frac{A'}{A}\partial_y + \frac{m^2}{A^2})\phi_t = M^2\phi_t - \frac{1}{\tilde{a}_t}O_t\tilde{a}_y\phi_y, \quad (23)$$

$$(-\partial_t^2 - 5\frac{\dot{a}_0}{a_0}\partial_t + \frac{\partial_i^2}{a_0^2} - 3[(\frac{\dot{a}_0}{a_0})^2 + \frac{\ddot{a}_0}{a_0}])\tilde{a}_t = m^2\tilde{a}_t, \quad (24)$$

$$O_t = 2\left\{\frac{\dot{a}_0}{a_0}A(4A' + A\partial_y) + A'A\partial_t\right\} \quad (25)$$

For A_i^L ,

$$(\partial_y^2 + 2\frac{A'}{A}\partial_y + \frac{m^2}{A^2})\phi_i^L = M^2\phi_i^L - \frac{2}{\tilde{a}_i^L}\partial_i(\frac{A'}{A}\tilde{a}_y\phi_y - \frac{\dot{a}_0}{a_0A^2}\tilde{a}_t\phi_t), \quad (26)$$

$$(-\partial_t^2 - \frac{\dot{a}_0}{a_0}\partial_t + \frac{\partial_i^2}{a_0^2})\tilde{a}_i^L = m^2\tilde{a}_i^L. \quad (27)$$

Here, we comment on the k -dependence of the above equations, which are written for $k = 0$ of solutions **(I)**. We consider also the solution **(II)** in which $k = -1$. It can be seen that the equations for $\phi_M(y)$ are not changed but ∂_i is changed by the three dimensional covariant derivative $\nabla_i(\gamma_{lm})$ in the other equations. And this changing gives no effect on the following analyses.

As is well known, the localization is determined by the equations of $\phi_M(y)$, and we do not need to know the details of $\tilde{a}_M(t, x^i)$. Although the equations for $\phi_t(y)$ and $\phi_i^L(y)$ include the inhomogeneous part, the homogeneous part of equations are equivalent to the one of ϕ_i^T . We can solve these equations for $\phi_t(y)$ and $\phi_i^L(y)$ by separating them into the special solutions, which satisfy the inhomogeneous equations, and the solutions, which satisfy the same form equation of (19), the homogeneous part. Then the general solutions are written as

$$\phi = \phi_{\text{special}} + \phi_i^T. \quad (28)$$

We expect that the localized modes are included in this general solutions through ϕ_i^T if A_i^T were trapped. Then the equations for $\phi_M(y)$ are separated to two groups of $A_\mu=(A_t, A_i)$ and A_y from the viewpoint of localization.

However A_y is an odd function with respect to y so we can say that $A_y|_{y=0} = 0$, then this component is not localized on the brane. While, we can see that the other components A_μ , which are parallel to the brane, are even functions of y from Eq.(17) which is written as,

$$\partial_A(\sqrt{-G}G^{AB}A_B) = \frac{1}{A^2}\partial_y(A^4A_y) - \frac{1}{a_0^3}\partial_t(a_0^3A_t) + \frac{1}{a_0^3}\partial_i(a_0A_i), \quad (29)$$

where we notice that the warp factor $A(y)$ is an even function of y . Then A_μ could be localized on the brane, and it is enough to examine only A_i^T for the problem of localization of the gauge boson since the homogeneous part of the equations of other components obeys the common equation with the one of A_i^T as stated above. So we concentrate on A_i^T hereafter.

Consider the equation (19) for A_i^T . For any solution of the background metric, $A(y)$ and $a_0(t)$, Eq.(19) can be rewritten in a one dimensional Schrödinger-type equation

$$[-\partial_z^2 + V(z)]u(z) = m^2u(z), \quad (30)$$

by introducing $u(z)$ and z defined as $\phi_i^T = A^{-1/2}u(z)$ and $\partial z/\partial y = \pm A^{-1}$. Here $A(y)$ is normalized as $A(0) = 1$ and the potential $V(z)$ is given as

$$V(z) = \frac{1}{4}(A')^2 + \frac{1}{2}AA'' + A^2M^2, \quad (31)$$

where z_0 represents the value of z at $y = 0$ and it depends on the solutions, $A(y)$, as well as $V(z)$. Here we should notice, $A' = \partial A/\partial y$ and the metric form (5).[¶]

The potentials are obtained for the three background solutions given in the previous section as follows,

(I-1) Solution for $\lambda > 0$ and $\Lambda < 0$:

$$V(z) = \frac{1}{4}\lambda\left[\frac{4M^2/\mu^2 + 3}{\sinh^2(\sqrt{\lambda}|z|)} + 1\right] - \frac{\kappa^2\tau}{6}\delta(|z| - z_0), \quad (32)$$

$$z = \text{sgn}(y)(\lambda)^{-1/2} \ln(\coth[\mu(y_H - |y|)/2])$$

$$z_0 = \frac{1}{\sqrt{\lambda}} \text{arcsinh}\left(\frac{\sqrt{\lambda}}{\mu}\right). \quad (33)$$

(I-2) Solution for $\lambda > 0$ and $\Lambda > 0$:

$$V(z) = \frac{1}{4}\lambda\left[\frac{4M^2/\mu_d^2 - 3}{\cosh^2(\sqrt{\lambda}z)} + 1\right] - \frac{\kappa^2\tau}{6}\delta(|z| - z_0), \quad (34)$$

$$z = \text{sgn}(y)(\lambda)^{-1/2} \ln(\cot[\mu_d(y_H - |y|)/2])$$

$$z_0 = \frac{1}{\sqrt{\lambda}} \text{arccosh}\left(\frac{\sqrt{\lambda}}{\mu_d}\right). \quad (35)$$

(II) Solution for $\lambda < 0$ and $\Lambda < 0$:

$$V(z) = \frac{1}{4}H^2\left[\frac{4M^2/\mu^2 + 3}{\cos^2(Hz)} - 1\right] - \frac{\kappa^2\tau}{6}\delta(|z| - z_0), \quad (36)$$

$$z = 2\text{sgn}(y) \arctan \tanh[\mu(y_H - |y|)/2]/H,$$

$$z_0 = \frac{1}{H} \arccos\left(\frac{H}{\mu}\right). \quad (37)$$

Before solving the above equation with the each potential obtained here, we should notice that only if the eigenvalue m^2 is smaller than the minimum of the potential except the δ function term which is denoted as V_{\min} , the corresponding field has the possibility to be trapped. For **(I)**, this minimum value is $V_{\min} = \lambda/4$. On the other hand for **(II)**, the minimum is given by $V_{\min} = M^2 + (3\mu^2 - H^2)/4$. The fluctuation with the mass $m^2 > V_{\min}$ corresponds to continuum mode.

Solution(I-1) First, we consider the case **(I-1)** where $\lambda = 0$ is realized when a fine-tuning condition between κ and $\Lambda < 0$ is satisfied as in the RS solution. The RS solution

[¶]When we use the notation, $A' = \partial A/\partial z$ and $ds^2 = e^{2\tilde{A}}\{-dt^2 + a^2(t)\gamma_{ij}(x^i)dx^i dx^j\} + dy^2$, we obtain $V(z) = \frac{1}{4}(\tilde{A}')^2 + \frac{1}{2}\tilde{A}'' + e^{2\tilde{A}}M^2$. This form would be more popular for some people.

would be obtained naturally when conformal invariance of the bulk is preserved for the brane-world solution [6]. Here this symmetry is not clear, so it will be natural to consider the general case of $\lambda \neq 0$. Among them, we firstly consider the case of $\lambda > 0$, **(I-1)**.

In this background, u is solved in terms of the following general solution, which is given by ignoring the δ -function potential,

$$u(z) = c_1 X^{-id} {}_2F_1(b_1, b_2; \bar{c}; -X) + c_2 X^{id} {}_2F_1(b'_1, b'_2; \bar{c}'; -X), \quad (38)$$

where c_1 and c_2 are constants of integration and

$$X = \frac{1}{\sinh^2(\sqrt{\lambda}z)}, \quad d = \frac{\sqrt{-1 + 4m^2/\lambda}}{4}, \quad (39)$$

$$b_1 = \frac{1}{4}(1 - \sqrt{1 + \hat{a}}) - id, \quad b_2 = \frac{1}{4}(1 + \sqrt{1 + \hat{a}}) - id, \quad \bar{c} = 1 - 2id, \quad (40)$$

$$b'_1 = \frac{1}{4}(1 - \sqrt{1 + \hat{a}}) + id, \quad b'_2 = \frac{1}{4}(1 + \sqrt{1 + \hat{a}}) + id, \quad \bar{c}' = 1 + 2id. \quad (41)$$

$$\hat{a} = 4M^2/\mu^2 + 3. \quad (42)$$

Here ${}_2F_1(\alpha, \beta; \gamma; x)$ denotes the Gauss's hypergeometric function. It follows from this solution that $u(z)$ oscillates with z when $m^2 > \lambda/4$, where the continuum KK modes appear.

Here we concentrate on the bound state which is restricted to the region of $m^2 < \lambda/4$. We note also $V_{\min} = \lambda/4$. In this case, d in (39) is rewritten as $d = i\bar{d}$, where \bar{d} is real and given below. We should notice this difference from the solution given in the KK mode region, and we need a converging solution in the region of the expected bound state. So the solution is obtained by setting $c_2 = 0$ and it is written as,

$$u(z) = c_1 X^{\bar{d}} {}_2F_1(b_1, b_2; \bar{c}; -X), \quad \bar{d} = \frac{\sqrt{1 - 4m^2/\lambda}}{4}, \quad (43)$$

which tends to zero in the limit of $z = \infty$ as $u(z) \rightarrow X^{\sqrt{1-4m^2/\lambda}/4}$. Then this solution is normalizable when $m^2 < \lambda/4$. One should notice here that we imposed one boundary condition at $z = \infty$ on the solution, which is written by two independent solutions.

The boundary condition at $z = z_0$ is given as follows by taking into account of the δ -function potential in (32),

$$\frac{du(z_0)}{dz} = -\frac{\kappa^2 \tau}{12} u(z_0). \quad (44)$$

From this setting we find the following results;

(1) Normalizability: The necessary condition of the localized state is the normalizability of this state, then the integration over y of the effective action for this mode

should be finite. It is easy to see that this is equivalent to demand the following condition for $u(z)$,

$$\int_{z_0}^{\infty} dz u(z)^2 < \infty. \quad (45)$$

As stated above this condition implies $m^2 < \lambda/4$ for our solution. We should notice that the solution for $M^2 = 0$ is excluded in the case of $\lambda = 0$ since here the solution is a constant. But this is not necessarily true in the case of $\lambda > 0$ since the solution is not a constant even if $M^2 = 0$. And the condition $m^2 < \lambda/4$ is a very loose condition which is equivalent to the condition of non KK-mode. The zero-mode, $m = 0$, of course satisfies this condition for $\lambda > 0$ even if how small it is. This fact is important since we don't need any bulk mass M for the vector field to obtain the normalizable zero-mode as in the case of RS brane of $\lambda = 0$ [8, 4]. In other words, any gauge bosons are trapped on the brane in a gauge invariant form when a positive cosmological constant λ exist.

In fact, we can see below that the zero mode eigenstate satisfies also the boundary condition (44) which is needed from the equation (30) for the precise form of potential (32).

(2) Relation between m^2 and M^2 : By solving (44), we find the M -dependence of the lowest eigenvalue m near $M^2 = 0$. It behaves as

$$m^2 = \alpha M^2, \quad (46)$$

where α depends on the other parameters λ and Λ . Here we can see $\alpha > 0$. This is confirmed both in numerically and analytically.

Firstly, we consider the eigenvalue m^2 analytically. In general, m^2 can be expressed as

$$m^2 = \int dz u^*(z) [-\partial_z^2 + V(z)] u(z) \equiv \langle u | [-\partial_z^2 + V(z)] | u \rangle, \quad (47)$$

where $u(z)$ denotes the normalized eigenfunction, $\langle u | u \rangle = 1$. Then we expand m^2 as

$$m^2 = m_0^2 + \alpha M^2 \quad (48)$$

for small M^2 , and α is expressed as

$$\alpha = \partial_{M^2} m^2|_{M^2=0} = \langle u | A^2 | u \rangle. \quad (49)$$

So we can see $\alpha > 0$. These points are assured by expanding the solution for m^2 obtained from Eq.(44) near an appropriate point. Here we examined m^2 near small λ and the following is found,

$$m^2 = \frac{1}{2} M^2. \quad (50)$$

This result implies $m_0 = 0$ for the solution **(I-1)**. Therefore we can say that zero-mode of the vector field is localized when its bulk mass M is zero. Then, all the gauge bosons without any gauge-symmetry breaking in the bulk are localized on the brane. This is completely different from the RS brane. And the massive bulk-vectors ($M^2 > 0$) are trapped as massive vectors ($m^2 > 0$) on the brane. In this sense, the situation is similar to the case of the graviton and scalar fields. In other words, the

general coordinate invariance in the bulk is also preserved on the dS brane as well as the gauge symmetry. The essential point here is the presence of positive λ , then we can say that the cosmological constant in our world is necessary to get a gauge symmetric theory in our universe even if how small it is. The numerical analysis is shown by the curve A in Fig.1, from which we can assure the statement given above.

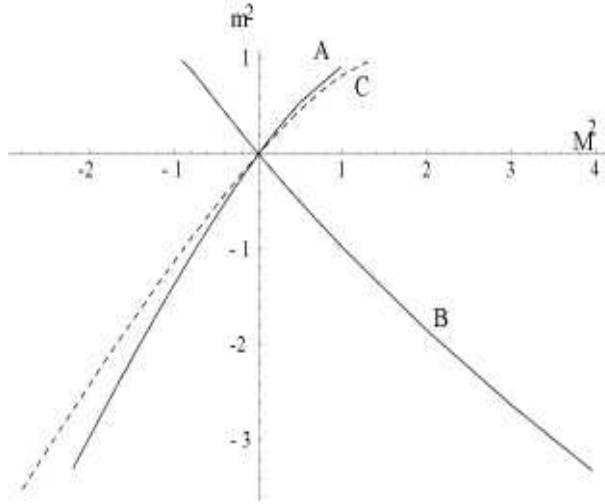


Fig. 1: The solid curves A and B show the eigenvalues m^2 versus M^2 for solutions (I) for $c = 0$ and $c = \hat{c}$ respectively, where $\lambda = \mu = 1$. The m^2 - M^2 curve for solution (II) is shown by dashed curve for $\lambda = 1.5$, $\mu = 1$. In both cases, m^2 is scaled by $\lambda/4$.

Solution(I-2) Next, we consider the solution (I-2). For this background, u is obtained in the same form given for the background (I-2) as

$$u(z) = c_1 Y^{-id} {}_2F_1(b_1, b_2; \bar{c}; Y) + c_2 Y^{id} {}_2F_1(b'_1, b'_2; \bar{c}'; Y), \quad (51)$$

where c_1 and c_2 are constants of integration. The same notations are used for the parameters in the solution, but X is replaced by

$$Y = \frac{1}{\cosh^2(\sqrt{\lambda}z)}, \quad (52)$$

and \hat{a} is changed to

$$\hat{a} = -4M^2/\mu_d^2 + 3. \quad (53)$$

For this solution, the normalizability condition is also satisfied if we choose $c_2 = 0$ as in the previous case. The boundary condition at $z = z_0$ can be written by the same formula (44) in terms of the first term of the solution given in (51). Then we perform the similar analysis as in the case of (I-1) to find the eigenvalue m^2 .

In this case, we can examine m^2 by expanding it near $M^2 = 0$ as $m^2 = m_0^2 + \alpha M^2$ as above, and we find $\alpha = \langle u|A^2|u \rangle$ is positive and $m^2 = (1/2)M^2$ near $\lambda = 0$ as in the case of solution (I-1). So $m_0 = 0$ also in this case. The example of an explicit numerical evaluation is shown in Fig.1 by the dashed curve. Then we can say that

the gauge bosons are trapped on the brane also in this brane solution. The important point is the existence of a positive λ , and the sign of Λ is not essential. This situation is also seen in the graviton trapping.

Solution(II) Finally, we consider the case of the background solution **(II)**. In this case the solution of the vector fluctuation (30) can be written as

$$u(z) = (1 - y)^{-\tilde{c}} v(y) , \quad (54)$$

$$v(z) = c_1 {}_2F_1(\tilde{b}_1, \tilde{b}_2; 1/2; y) + c_2 y^{1/2} {}_2F_1(\tilde{b}_1 + 1/2, \tilde{b}_2 + 1/2; 3/2; y) , \quad (55)$$

$$y = \sin^2(Hz) , \quad \tilde{c} = -\frac{1}{4} + \frac{1}{2}\sqrt{1 + M^2/\mu^2} , \quad (56)$$

$$\tilde{b}_1 = -\tilde{c} - \frac{1}{4}\sqrt{1 + 4m^2/H^2}, \quad \tilde{b}_2 = -\tilde{c} + \frac{1}{4}\sqrt{1 + 4m^2/H^2}, \quad (57)$$

where c_1 and c_2 are constants of integration.

In this case, the potential $V(z)$ diverges at $z_1 = \pi/(2H)$. Then we need a new boundary condition at z_1

$$u(z_1) = 0 \quad (58)$$

in addition to the condition (44) at $z = z_0$. We solve these two conditions as follows. The condition (44) is used to fix the ratio c_2/c_1 as

$$c_2/c_1 = -\frac{B_0 F(y_0) + F'(y_0)}{B_0 G(y_0) + G'(y_0)} \quad (59)$$

where $F(y) = {}_2F_1(\tilde{b}_1, \tilde{b}_2; 1/2; y)$, $G(y) = y^{1/2} {}_2F_1(\tilde{b}_1 + 1/2, \tilde{b}_2 + 1/2; 3/2; y)$, $y_0 = \sin^2(Hz_0)$ and $' = \partial/\partial y$. The factor B_0 is expressed by

$$B_0 = \frac{1}{2} \left(\frac{\mu}{H} \right)^2 \sqrt{1 + \frac{M^2}{\mu^2}} , \quad (60)$$

The condition (58) can be written as,

$$c_2/c_1 = -2 \frac{\Gamma(1 - \tilde{b}_1) \Gamma(1 - \tilde{b}_2)}{\Gamma(1/2 - \tilde{b}_1) \Gamma(1/2 - \tilde{b}_2)} . \quad (61)$$

Using above two conditions (59) and (61), we can obtain the eigenvalue m^2 as a value dependent on M^2 and other parameters. When we express the M^2 dependence of m^2 as the form $m^2 = m_0^2 + \alpha M^2$, we find the following,

$$\alpha = \langle u | A^2 | u \rangle + \frac{\partial u(z_1)}{\partial z} \frac{\partial u(z_1)}{\partial M^2} . \quad (62)$$

Then α is not necessarily positive. The numerical results are shown by the dashed lines in Fig.2. In this case, m_0^2 depends on H and becomes zero for specific H . Furthermore H has the upper bound so as to be $m_0^2 < V_{\min}$, which is condition to be trapped at $M = 0$. The relation between m_0^2 and H is shown in Fig.3. The upper bound to H is the value for the point shown by \oplus in the large μ region corresponding to curve A. When μ takes a small value below the value corresponding to B, the gauge field, that

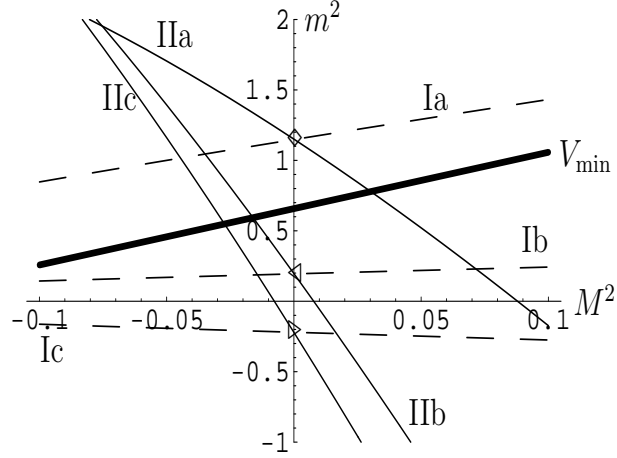


Fig. 2: The dashed curves show the m^2 versus M^2 curves for $c = 0$, Ia, Ib and Ic for $H = 0.5, 0.41, 0.4$, respectively. The curves for $c = \hat{c}$ are shown by the solid lines IIa, IIb and IIc for $H = 0.5, 0.41, 0.4$, respectively. The points at $M = 0$ for each lines are denoted by \diamond , \triangleleft , \triangleright . The curve V_{\min} is written for $H = 0.5$. In all cases, m^2 and V_{\min} are scaled by H^2 , where $\mu = 0.55$.

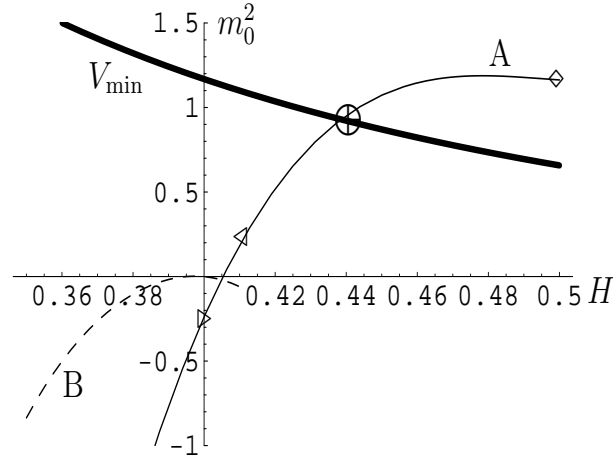


Fig. 3: The curves A and B show the eigenvalues m_0^2 versus H for the parameters $\mu = 0.55, 0.422$, respectively. Both lines have end in right hand side to satisfy $H < \mu$. Also the points corresponding in Fig.2 are shown by \diamond , \triangleleft , \triangleright . The point for $m_0^2 = V_{\min}$ is denoted by \oplus . $m_0^2 = 0$ is realized for $H = 0.405$. V_{\min} is written for $\mu = 0.55$. In all cases, m_0^2 and V_{\min} are scaled by H^2 .

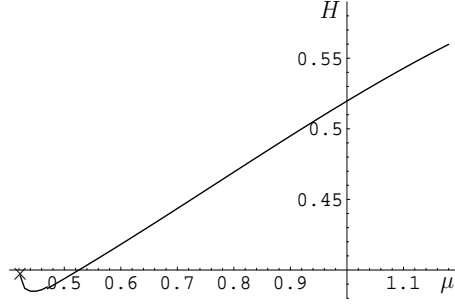


Fig. 4: H - μ curve with $m_0 = M = 0$. The smallest value for H is 0.385 and the smallest value for μ is 0.422. The solution does not exist for $\mu < 0.422$.

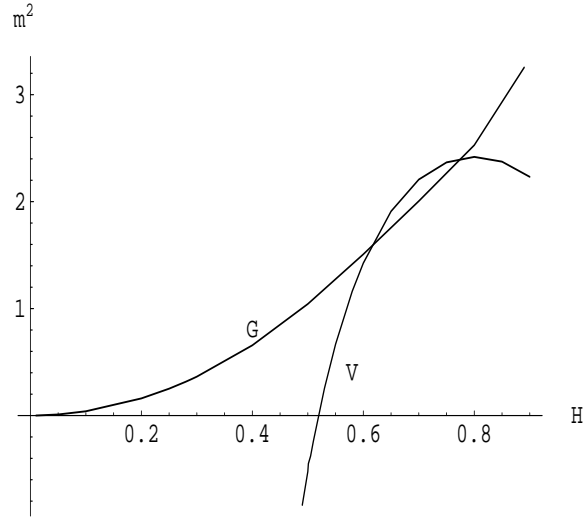


Fig. 5: The value m satisfied the boundary conditions are shown for the graviton (G) and gauge boson (V) versus H for $\mu = 1$.

is, the fluctuation with $m_0 = 0$ is not trapped for $M = 0$. So μ has the lower bound for the gauge field to be trapped with $M = 0$. The curve which H and μ draw for $m_0 = M = 0$ is shown in Fig.4. H also has the lower bound. This means that the bulk gauge field is not trapped as gauge field for very small H not only in the case of $\lambda = 0$ as previously known.

It should be astonishing that the gauge bosons can be trapped for $\lambda < 0$ differently from the case of the graviton which can not be trapped in this case. To make clear the difference of gauge bosons and the graviton, we show here the lowest mass of the trapped modes for both particles in Fig.5. For graviton, m^2 is not able to become zero since $H \neq 0$. This behaviour is also shown in [3]. While m^2 of gauge boson crosses zero at finite H as shown above for different parameters. So there is a point in the parameter-space where gauge bosons are trapped.

4 Vector localization: the case of $c = \hat{c}$

Here we examine the vector localization for $c = \hat{c}$ case. As mentioned above, the analysis is similar to the case of $c = 0$ in the previous section. Here the field equation of A_M is given as

$$\frac{1}{\sqrt{-G}}\partial_A[\sqrt{-G}(G^{AB}G^{CD} - G^{AC}G^{BD})\partial_B A_C] - [M^2 + \hat{c}\delta(y)]G^{DB}A_B = 0. \quad (63)$$

The δ function term appears newly. Because of this term, the identity is presented by

$$\partial_A([M^2 + \hat{c}\delta(y)]\sqrt{-G}G^{AB}A_B) = 0. \quad (64)$$

However the equation (64) can reduce to Eq.(17) as follows. Consider near $y = 0$ of the equation (63) for $D = y$, and integrate it over y for the region $-\epsilon < y < \epsilon$, where ϵ is an infinitesimally small number. Then we find $A_y(x, 0) = 0$ which implies $A_y(x, y)$ is odd with respect to the reflection $y \rightarrow -y$. Then we can write (64) as,

$$M^2\partial_A(\sqrt{-G}G^{AB}A_B) + \hat{c}\delta(y)\partial_\mu(\sqrt{-G}G^{\mu\nu}A_\nu) = 0. \quad (65)$$

Integrating this again in the region $-\epsilon < y < \epsilon$, we find $\partial_\mu(\sqrt{-G}G^{\mu\nu}A_\nu)|_{y=0} = 0$. As a result, the identity (64) can be written as

$$\partial_A(\sqrt{-G}G^{AB}A_B) = 0, \quad (66)$$

at any value of y . So we can solve the above field equation by using (66). The expansion of the field equation in terms of four-dimensional mass eigenstates leads only to replace M^2 with $M^2 + c\delta(y)$ in Eqs.(19)-(27).

Again we need only to consider the equation for A_i^T . Then the Schrödinger type equation (30) has the potential

$$V(z) = \frac{1}{4}(A')^2 + \frac{1}{2}AA'' + A^2M^2 + \hat{c}\delta(|z| - z_0) \quad (67)$$

where $\Lambda < 0$ is considered by the definition of \hat{c} . The explicit potentials are given as follows,

(I-1) Solution for $\lambda > 0$ and $\Lambda < 0$:

$$V(z) = \frac{1}{4}\lambda \left[\frac{4M^2/\mu^2 + 3}{\sinh^2(\sqrt{\lambda}|z|)} + 1 \right] - \left(\frac{\kappa^2\tau}{6} - \hat{c} \right) \delta(|z| - z_0), \quad (68)$$

(II) Solution for $\lambda < 0$ and $\Lambda < 0$:

$$V(z) = \frac{1}{4}H^2 \left[\frac{4M^2/\mu^2 + 3}{\cos^2(Hz)} - 1 \right] - \left(\frac{\kappa^2\tau}{6} - \hat{c} \right) \delta(|z| - z_0), \quad (69)$$

Away from the δ -function term, the bound state solutions are given by Eqs.(43) and (54) respectively. On the other hand, the boundary condition at $z = z_0$ is changed as

$$\frac{du(z_0)}{dz} = -\frac{1}{2} \left(\frac{\kappa^2\tau}{6} - \hat{c} \right) u(z_0). \quad (70)$$

The results of our analyses are summarized as follows for the above two solutions.

For Solution (I-1): In this case the consideration on the normalizability yields the only loose condition $\mu^2 < \lambda/4$ as in the case of $c = 0$. However the relation between m^2 and M^2 is noticeably different. For this case with the brane coupling, α in Eq.(46) is negative. This is also confirmed both in numerically and analytically.

From the general representation (47), α is expressed for small M^2 as

$$\alpha = \partial_{M^2} m^2|_{M^2=0} = \langle u|A^2|u \rangle - \frac{1}{\mu} |u(z_0)|^2, \quad (71)$$

where the second term in (71) is coming from the term \hat{c} in the potential (68). So we expect that α might be negative. In fact when we examine m^2 near small λ , the following relation is obtained,

$$m^2 = -\frac{\lambda}{4\mu^2} M^2. \quad (72)$$

This implies that the massive bulk vector is trapped as a tachyonic vector on the brane when $\lambda > 0$ differently from the case of $\lambda = 0$. Then this brane solution is unstable. It would be possible to improve this instability by considering a dynamical scenario that the trapped tachyonic vector would condense to change the brane-tension τ to a small value until arriving at the stable point $\lambda = 0$. The λ -dependence in Eq.(72) seems to support this scenario. So we can say that the case of $c = \hat{c}$ gives a brane where the cosmological constant is always zero due to the gauge invariance on the brane. The numerical analysis is shown by the curve B in Fig.1.

For Solution (II): In this case, the factor B_0 in Eq.(59) is given instead of Eq.(60) as

$$B_0 = \frac{1}{2} \left(\frac{\mu}{H} \right)^2 \left(\sqrt{1 + \frac{M^2}{\mu^2}} + \frac{\sqrt{1 + \frac{M^2}{\mu^2}} - 1}{\sqrt{1 - \frac{H^2}{\mu^2}}} \right), \quad (73)$$

where $H < \mu$ since $\mu^2 - H^2 = \kappa^4\tau^2/36$. Then α is obtained as

$$\alpha = \langle u|A^2|u \rangle + \frac{\partial u(z_1)}{\partial z} \frac{\partial u(z_1)}{\partial M^2} - \frac{1}{\mu} |u(z_0)|^2. \quad (74)$$

The numerical estimation is shown by solid lines in Fig.2. This implies also that massive bulk vector is trapped as a tachyonic vector on the brane.

5 Summary

We have found the possibility of the gauge fields trapping on the brane through the analyses given here for vector fields. We have examined the localization of the fields on both dS and AdS brane or on positive and negative λ . The bulk cosmological constant Λ is also considered for both negative and positive cases. The model considered here include the coupling of vectors and the brane through the mass term of the vector with its coupling c .

For the case of $c = 0$, the brane model is normal and includes no special brane coupling, the bulk gauge bosons can be trapped on the dS brane, i.e. for the brane of positive λ , for both cases of positive and negative Λ . Then, we can say that the gauge symmetries in the bulk theory are preserved also on the dS brane. This situation is similar to the case of the graviton localization. Namely the general coordinate invariance in the bulk is also preserved on the dS brane due to the graviton trapping.

The difference between the graviton and the gauge field appears for $\lambda = 0$, the RS brane. Actually, the gauge bosons can not be trapped on the RS brane since the cosmological constant is zero. Then we needed an appropriate coupling c of gauge bosons and the brane for its trapping on the $\lambda = 0$ brane as shown above.

For non-zero c , the bulk mass of the vector field was needed to trap the vector fields on the brane as a gauge boson, i.e. as a massless 4d vector. When this model is applied to the case of dS brane ($\lambda > 0$), we find an unexpected phenomena that the bulk massive vectors are trapped on the dS brane as tachyonic vectors. Then the dS brane is unstable in this model.

In this case however, the bulk gauge symmetries is broken in spite of the gauge symmetries on the brane. One possible idea to change this situation is to consider a model which contains the dilaton. And our effective action of massive vector fields can be obtained as an effective action by an appropriate field transformation [11] from the original gauge invariant theory. If this is true, then the trapping of the gauge bosons will be solved in terms of the the dilaton theory with bulk gauge symmetry. On this point, we will discuss in a separate paper.

Next, we summarize for the AdS brane ($\lambda < 0$). In this case, the graviton can not be trapped as a massless mode on the brane. However we find that the bulk-vector bosons can be trapped on the brane as gauge bosons for both models $c = 0$ and $c = \hat{c}$. This is realized for a specific value of $|\lambda|$, then we need a fine-tuning.

Finally, we comment on the gap or the lower bound for the continuum KK mode for the dS brane. It starts from $m^2 = \lambda/4$. As a result, massive vector can be trapped in the range, $m^2 < \lambda/4$, and its value is related to the bulk mass M^2 as $m^2 = \alpha M^2$ where α is positive and dependent on the parameters of the theory. We can expect that these localized fields of small mass vectors would play some role in the cosmological scenario if they were really exist.

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